Multiple Regression

Simple Regression:-

- Study of 2 variables (one is independent other is dependent variable)
- Used for prediction of change of dependent variables according to change of Independent variables.

Regression $(Y) = a + bX$

Y — dependent

X — Independent

exp

Situation	Independent variable		Dependent variable	
	Variable	Cause	Variable	effect
Case-1	Price	50	Sales	5,000
Case-2	Price	60	Sales	4,000
Case-3	Price	40	Sales	6,000
Case-4	Price	55	Sales	?
Case-5	Price	35	Sales	7,000

Multiple Regression

- Many variables (More than 2)

① Dependent variable — Many Independent variable

- Prediction of change of dependent variable in accordance of change in Independent variables.

ex —

	Price (Ind)	Advt (Ind)	Sales (Dep)
Case-1	50	2 mn	50 mn
Case-2	40	220 mn	60 mn
Case-3	60	150 mn	40 mn
" 4	55	210 mn	?
" 5	?	2 mn	750 mn
" 6	45	?	650 mn

* Methods of Multiple Regression

① Least square Method (direct)

③ Least square method using mean ($\bar{x}$) / short cut method)

②

(a) Least Square value using mean (different method) (when $X_1 X_2 X_3$ data are given)

(of X_1 on X_2 & X_3)

$$X_1 - \bar{X}_1 = b_{12\cdot3}(X_2 - \bar{X}_2) + b_{13\cdot2}(X_3 - \bar{X}_3)$$

where $x_1 = (X_1 - \bar{X}_1)$, $x_2 = (X_2 - \bar{X}_2)$

$x_3 = (X_3 - \bar{X}_3)$

or $x_1 = b_{12\cdot3}\, x_2 + b_{13\cdot2}\, x_3$

where $b_{12\cdot3}$ & $b_{13\cdot2}$ = partial regression co-efficients

* The values of partial regression co-efficients can be obtained by solving the following two normal equations.

$$\Sigma x_1 x_2 = b_{12\cdot3} \Sigma x_2^2 + b_{13\cdot2} \Sigma x_1 x_2$$

$$\Sigma x_1 x_3 = b_{12\cdot3} \Sigma x_2 x_1 + b_{13\cdot2} \Sigma x_3^2$$

further solved

$$b_{12\cdot3} = \frac{\{(\Sigma x_1 x_2)(\Sigma x_3)^2\} - [(\Sigma x_1 x_3)(\Sigma x_2 x_3)]}{(\Sigma x_2^2)(\Sigma x_3^2) - (\Sigma x_2 x_3)^2} \quad \text{---(i)}$$

(1)

$$b_{13.2} = \frac{\left[(\Sigma x_1 x_3)(\Sigma x_2^2)\right] - \left[(\Sigma x_1 x_2)(\Sigma x_3 x_2)\right]}{(\Sigma x_3^2)(\Sigma x_2^2) - (\Sigma x_3 x_2)^2}$$

* If X_2 on X_1 & X_3

$$(X_2 - \bar{X}_2) = b_{21.3}(X_1 - \bar{X}_1) + b_{23.1}(X_3 - \bar{X}_3)$$

or $x_2 = b_{21.3} x_1 + b_{23.1} x_3$

where $b_{21.3}$ & $b_{23.1}$ = partial regression coefficient.

$$b_{21.3} = \frac{(\Sigma x_2 x_1)(\Sigma x_3^2) - (\Sigma x_2 x_3)(\Sigma x_1 x_3)}{(\Sigma x_1^2)(\Sigma x_3^2) - (\Sigma x_1 x_3)^2}$$

$$b_{23.1} = \frac{(\Sigma x_2 x_3)(\Sigma x_1^2) - (\Sigma x_2 x_1)(\Sigma x_3 x_1)}{(\Sigma x_3^2)(\Sigma x_1^2) - (\Sigma x_3 x_1)^2}$$

* If X_3 on X_1 & X_2

$$(X_3 - \bar{X}_3) = b_{31\cdot2}(X_1 - \bar{X}_1) + b_{32\cdot1}(X_2 - \bar{X}_2)$$

$$\Rightarrow x_3 = b_{31\cdot2}\, x_1 + b_{32\cdot1}\, x_2$$

where $b_{31\cdot2}$ & $b_{32\cdot1}$ are partial regression co-effn

$$b_{31\cdot2} = \frac{(\Sigma x_3 x_1)(\Sigma x_2^2) - (\Sigma x_3 x_2)(\Sigma x_1 x_2)}{(\Sigma x_1^2)(\Sigma x_2^2) - (\Sigma x_1 x_2)^2}$$

&

$$b_{32\cdot1} = \frac{(\Sigma x_3 x_2)(\Sigma x_1^2) - (\Sigma x_3 x_1)(\Sigma x_2 x_1)}{(\Sigma x_2^2)(\Sigma x_1^2) - (\Sigma x_2 x_1)^2}$$

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ex- find the least square regression of X_3 on X_1 & X_2 using Actual mean method. Also estimate X_3, when $X_1 = 10$ & $X_2 = 6$

X_1	3	5	6	8	12	14
X_2	16	10	7	4	3	2
X_3	90	72	54	42	30	12

X_1	x_1 $(X_1-\bar{X}_1)$	x_1^2	X_2	x_2	x_2^2	X_3	x_3	x_3^2	x_1x_2	x_1x_3	x_2x_3
3	−5	25	16	9	81	90	40	1600	−45	−200	360
5	−3	9	10	3	9	72	22	484	−9	−66	66
6	−2	4	7	0	0	54	04	16	0	−8	0
8	0	0	4	−3	9	42	−8	64	0	0	24
12	4	16	3	−4	16	30	−20	400	−16	−80	80
14	6	36	2	−5	25	12	−38	1444	−30	−228	190
$\Sigma X_1 = 48$	$\Sigma x_1 = 0$	$\Sigma x_1^2 = 90$	$\Sigma X_2 = 42$	$\Sigma x_2 = 0$	$\Sigma x_2^2 = 140$	$\Sigma X_3 = 300$	$\Sigma x_3 = 0$	$\Sigma x_3^2 = 4008$	$\Sigma x_1x_2 = -100$	$\Sigma x_1x_3 = -582$	$\Sigma x_2x_3 = 720$

$\bar{X}_1 = \frac{\Sigma X_1}{n_1} = 48/6 = 8$ / $\bar{X}_2 = \Sigma X_2/n_2 = 42/6 = 7$ / $\bar{X}_3 = \Sigma X_3/n_3 = 300/6 = 50$

If X_3 on x_1 & x_2

$\Rightarrow (X_3 - \bar{X}_3) = b_{31\cdot 2}(X_1 - \bar{X}) + b_{32\cdot 1}(X_2 - \bar{X}_2)$

$\Rightarrow x_3 = b_{31\cdot 2}\, x_1 + b_{32\cdot 1}\, x_2$

$\therefore\ b_{31\cdot 2} = \dfrac{(\Sigma x_3 x_1)(\Sigma x_2^2) - (\Sigma x_3 x_2)(\Sigma x_1 x_2)}{(\Sigma x_1^2)(\Sigma x_2^2) - (\Sigma x_1 x_2)^2}$

$= \dfrac{(-582 \times 140) - [(720 \times (-100))]}{(90 \times 140) - (-100 \times -100)}$

$= (-81480 + 72000) / (12600 - 10000)$

$= -9480 / 2600 = -3.65$ //

$\therefore\ b_{32\cdot 1} = \dfrac{(\Sigma x_3 x_2)(\Sigma x_1^2) - (\Sigma x_3 x_1)(\Sigma x_2 x_1)}{(\Sigma x_2^2)(\Sigma x_1^2) - (\Sigma x_2 x_1)^2}$

$= \dfrac{(720 \times 90) - (-582 \times -100)}{(140 \times 90) - (-100 \times -100)}$

$= \dfrac{64800 - 58200}{12600 - 10000} = 6600/2600$

$= 2.54$ //

Accarding to Qn, $X_1 = 10$, $X_2 = 6$
what is X_3 (?)

$$X_3 - \bar{X}_3 = b_{31\cdot 2}(X_1 - \bar{X}_1) + b_{32\cdot 1}(X_2 - \bar{X}_2)$$

$$\Rightarrow (X_3 - 50) = -3\cdot 65(X_1 - 8) + 2\cdot 54(X_2 - 7)$$

$$\Rightarrow X_3 = -3\cdot 65X_1 + 2\cdot 54X_2 + 61\cdot 4$$

If $X_1 = 10$ & $X_2 = 6$, then

$$X_3 = (-3\cdot 65 \times 10) + (2\cdot 54 \times 6) + 61\cdot 4$$

$$= 40$$

Ans

exp

X_1 on X_2 & X_3

X_1	4	6	7	9	13	15
X_2	15	12	8	6	4	3
X_3	30	24	20	14	10	4

X_1	x_1 $(X_1-\bar{X}_1)$	x_1^2	X_2	x_2 $(X_2-\bar{X}_2)$	x_2^2	X_3	x_3 $(X_3-\bar{X}_3)$	x_3^2	x_1x_2	x_1x_3	x_2x_3
4	-5	25	15	7	49	30	13	169	-35	-65	91
6	-3	9	12	4	16	24	7	49	-12	-21	28
7	-2	4	8	0	0	20	3	9	0	-6	0
9	0	0	6	-2	4	14	-3	9	0	0	6
13	4	16	4	-4	16	10	-7	49	-16	-28	28
15	6	36	3	-5	25	4	-13	169	-30	-78	65
$\Sigma X_1=$ 54	$\Sigma x_1=$ 0	$\Sigma x_1^2=$ 90	$\Sigma X_2=$ 48	$\Sigma x_2=$ 0	$\Sigma x_2^2=$ 110	$\Sigma X_3=$ 102	$\Sigma x_3=$ 0	$\Sigma x_3^2=$ 454	$\Sigma x_1x_2=$ -93	$\Sigma x_1x_3=$ -198	$\Sigma x_2x_3=$ 218

$\bar{X}_1 = \Sigma X_1/n_1 = 54/6 = 9$ | $\bar{X}_2 = \Sigma X_2/n_2 = 48/6 = 8$ | $\bar{X}_3 = \Sigma X_3/n_3 = 102/6 = 17$

If X_1 on X_2 & X_3 $(X_2 = 20, X_3 = 11)$ what is X_1?

$\Rightarrow (X_1-\bar{X}_1) = b_{12\cdot3}(X_2-\bar{X}_2) + b_{13\cdot2}(X_3-\bar{X}_3)$ — (i)

$\Rightarrow x_1 = b_{12\cdot3}\, x_2 + b_{13\cdot2}\, x_3$ ....

$$\Rightarrow b_{12\cdot3} = \frac{(\Sigma x_1x_2)(\Sigma x_3^2) - (\Sigma x_1x_3)(\Sigma x_2x_3)}{(\Sigma x_2^2)(\Sigma x_3^2) - (\Sigma x_2x_3)^2}$$

$$= \frac{(-93 \times 454) - (-198 \times 218)}{(110 \times 454) - (218 \times 218)}$$

$$= \left[(-42222) + (43164)\right] / (49940 - 47524)$$

$$= 942 / 2416 = 0.39$$

$$\Rightarrow b_{13\cdot2} = \frac{(\Sigma x_1x_3)(\Sigma x_2^2) - (\Sigma x_1x_2)(\Sigma x_3x_2)}{(\Sigma x_3^2)(\Sigma x_2^2) - (\Sigma x_3x_2)^2}$$

$$= \frac{\left[(-198) \times 110\right] - \left[(-93) \times (218)\right]}{(454 \times 110) - (218 \times 218)}$$

$$= \frac{(-21780) + 20274}{49940 - 47524} = \frac{-1506}{2416}$$

$$= -0.62$$

$X_2 = 20, \; X_3 = 11, \; X_1 = ?$

$$X_1 - \bar{X}_1 = b_{12\cdot3}(X_2 - \bar{X}_2) + b_{13\cdot2}(X_3 - \bar{X}_3)$$

$$\Rightarrow X_1 - 9 = 0\cdot39(20-8) + (-0\cdot62)(11-17)$$

$$X_1 = 4\cdot68 + 3\cdot72 + 9 = 17\cdot4$$